

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 1	Daily Tutorial Sheet - 6
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76.(B) $\cos \theta = \frac{1}{2} \left(x + \frac{1}{x} \right)$

$$x + \frac{1}{x} \geq 2 \text{ or } x + \frac{1}{x} \leq -2 \Rightarrow \cos \theta \geq 1 \text{ or } \cos \theta \leq -1$$

$$\Rightarrow \cos \theta = x = \pm 1 \Rightarrow x^2 = 1 \text{ and } \sin \theta = 0 \Rightarrow \frac{1}{2} \left(x^2 + \frac{1}{x^2} \right) = 1$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 = \sec 2\theta$$

$$\sin 2\theta = \tan 2\theta = 0$$

77.(B) $\sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} + \sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}} = \sqrt{\frac{1+\tan x}{1-\tan x}} + \sqrt{\frac{1-\tan x}{1+\tan x}} = \frac{2}{\sqrt{1-\tan^2 x}} = \frac{2\sqrt{\cos^2 x}}{\sqrt{\cos^2 x - \sin^2 x}} = \frac{2|\cos x|}{\sqrt{\cos 2x}}$

78.(D) Observe : $\frac{2\pi}{5} - \frac{\pi}{15} = \frac{\pi}{3} \Rightarrow \frac{\tan\left(\frac{2\pi}{5}\right) - \tan\left(\frac{\pi}{15}\right)}{1 + \tan\left(\frac{2\pi}{5}\right)\tan\left(\frac{\pi}{15}\right)} = \tan \frac{\pi}{3} = \sqrt{3}$

$$\Rightarrow \tan\left(\frac{2\pi}{5}\right) - \tan\left(\frac{\pi}{15}\right) - \sqrt{3} \tan\left(\frac{2\pi}{5}\right)\tan\left(\frac{\pi}{15}\right) = \sqrt{3}$$

79.(A) Let $\tan(\alpha + 2\beta)\tan(2\alpha + \beta) = \frac{\sin(\alpha + 2\beta)\sin(2\alpha + \beta)}{\cos(\alpha + 2\beta)\cos(2\alpha + \beta)} = k$

Adding (-1) on both sides

$$\frac{\sin(\alpha + 2\beta)\sin(2\alpha + \beta) - \cos(\alpha + 2\beta)\cos(2\alpha + \beta)}{\cos(\alpha + 2\beta)\cos(2\alpha + \beta)} = k - 1$$

$$\Rightarrow \frac{-\cos(3(\alpha + \beta))}{\cos(\alpha + 2\beta)\cos(2\alpha + \beta)} = k - 1$$

$$\sin(\alpha + \beta) = 1 \Rightarrow \cos(\alpha + \beta) = 0$$

$$\Rightarrow \cos(3(\alpha + \beta)) = \cos(\alpha + \beta) [4\cos^2(\alpha + \beta) - 3] = 0$$

$$\text{Hence, } k - 1 = 0 \Rightarrow k = 1$$

80.(B) $25\cos^2 \theta + 5\cos \theta - 12 = 0$

$$\Rightarrow \cos \theta = \frac{-5 \pm \sqrt{25 + 4 \times 25 \times 12}}{50} = \frac{-5 \pm 35}{50} = \frac{-4}{5}, \frac{3}{5}$$

$$\cos \theta = \frac{-4}{5} \text{ as } \frac{\pi}{2} < \alpha < \pi \Rightarrow \sin \theta = \frac{3}{5} \Rightarrow \sin 2\theta = 2\sin \theta \cos \theta = \frac{-24}{25}$$

81.(A) $\sin(\theta - x) = a$ and $\cos(\theta - y) = b$

$$\Rightarrow \cos(\theta - x) = \pm \sqrt{1 - a^2} \text{ and } \sin(\theta - y) = \pm \sqrt{1 - b^2}$$

$$\Rightarrow \cos(x-y) = \cos((\theta-y) - (\theta-x)) = \cos(\theta-y)\cos(\theta-x) + \sin(\theta-x)\sin(\theta-y) = \pm b\sqrt{1-a^2} \pm a\sqrt{1-b^2}$$

$$\Rightarrow \cos(x-y) = \pm b\sqrt{1-a^2} \pm a\sqrt{1-b^2}$$

$$\Rightarrow \cos(x-y) \text{ can take 4 possible values at most.}$$

(A) option is one of those values.

82.(D) $\sin x + \cos y = a$ and $\cos x + \sin y = b$

$$\Rightarrow a - b = \sin x - \sin y + \cos y - \cos x = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2} + 2 \sin \left(\frac{x-y}{2} \right) \sin \left(\frac{x+y}{2} \right)$$

$$\Rightarrow a + b = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} + 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\Rightarrow \frac{a-b}{a+b} = \frac{2 \sin \left(\frac{x-y}{2} \right) \left[\cos \frac{x+y}{2} + \sin \frac{x+y}{2} \right]}{2 \cos \frac{x-y}{2} \left[\sin \frac{x+y}{2} + \cos \frac{x+y}{2} \right]} = \tan \left(\frac{x-y}{2} \right)$$

83.(A) $\sin \theta + \operatorname{cosec} \theta \geq 2$ and $\sin \theta + \operatorname{cosec} \theta \leq -2$

For $\sin \theta + \operatorname{cosec} \theta = 2$; $\sin \theta = 1 = \operatorname{cosec} \theta \Rightarrow \sin^8 \theta + \operatorname{cosec}^8 \theta = 1^8 + 1^8 = 2$

84.(B) $x = \frac{-y}{2} = \frac{-z}{2} = k$; $xy + yz + zx = -2k^2 + 4k^2 + (-2k^2) = 0$

85.(C) We have $\sin \theta + \sin^2 \theta = 1 \Rightarrow \sin \theta = 1 - \sin^2 \theta \Rightarrow \sin \theta = \cos^2 \theta$

Now, $\cos^2 \theta + \cos^4 \theta = \cos^2 \theta + (\cos^2 \theta)^2 \Rightarrow \cos^2 \theta + \cos^4 \theta = \cos^2 \theta + \sin^2 \theta \Rightarrow \cos^2 \theta + \cos^4 \theta = 1$

86.(B) $\cos \left(x - \frac{3\pi}{2} \right) + \sin \left(\frac{3\pi}{2} + x \right) + \sin(32\pi + x) - 18 \cos(19\pi - x) + \cos(56\pi + x) - 9 \sin(x + 17\pi)$
 $= -\sin x - \cos x + \sin x + 18 \cos x + \cos x + 9 \sin x = 18 \cos x + 9 \sin x$
 $a + b = 27$

87.(C) $\cos 2001\pi + \cot 2001 \frac{\pi}{2} + \sec \frac{2001\pi}{3} + \tan \frac{2001\pi}{4} + \operatorname{cosec} \frac{2001\pi}{6}$
 $= \cos \pi + \cot \frac{\pi}{2} + \sec 667\pi + \tan \frac{\pi}{4} + \operatorname{cosec} \frac{667\pi}{2} = -1 + 0 - 1 + 1 - 1 = -2$

88.(D) $\cot 123^\circ \cot 133^\circ \cot 137^\circ \cot 147^\circ$
 $= \cot(90 + 33) \cot(90 + 43) \cot(180 - 43) \cot(180 - 33) = (-\tan 33)(-\tan 43)(-\cot 43)(-\cot 33) = 1$

89.(ABCD) $\sec A = \frac{17}{8}$, $\operatorname{cosec} B = \frac{5}{4}$; $\cos A = \frac{8}{17}$, $\sin B = \frac{4}{5}$; $\sin A = \pm \frac{15}{17}$, $\cos B = \pm \frac{3}{5}$

$$\sec(A+B) = \frac{1}{\cos A \cos B - \sin A \sin B} = \frac{1}{\pm \frac{8}{17} \times \pm \frac{3}{5} \pm \frac{4}{5} \times \frac{15}{17}} = \frac{-85}{36}, \frac{85}{36}, -\frac{85}{84}, \frac{85}{84}$$

$$\begin{aligned}
 \text{90.(A)} \quad 2 \tan \frac{\alpha}{2} = \tan \frac{\beta}{2}; \quad \frac{3+5 \cos \beta}{5+3 \cos \beta} &= \frac{3+5 \left(\frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}{5+3 \left(\frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)} = \frac{3 \left(1 + \tan^2 \frac{\beta}{2} \right) + 5 \left(1 - \tan^2 \frac{\beta}{2} \right)}{5 \left(1 + \tan^2 \frac{\beta}{2} \right) + 3 \left(1 - \tan^2 \frac{\beta}{2} \right)} \\
 &= \frac{8 - 2 \tan^2 \frac{\beta}{2}}{8 + 2 \tan^2 \frac{\beta}{2}} = \frac{8 - 8 \tan^2 \frac{\alpha}{2}}{8 + 8 \tan^2 \frac{\alpha}{2}} = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \cos \alpha
 \end{aligned}$$